Maximizing success when it is the product of two things that go in opposite directions: the magic of elasticity

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In this third article in a series on health economics, we focus on maximizing outcomes when leaders must decide how to choose the right way to balance factors that are inversely related, like price and quantity. Building on the previous articles' themes of efficiency and costs, this article focuses on a concept called elasticity and shows how it can provide leaders with intuition about whether they should scale down or rachet up their efforts. Knowledge of elasticity can help leaders successfully identify optimal courses of action.

KEY WORDS: leadership, optimization, elasticity, health economics

Many things are inversely related. Statisticians describe such relations between two variables as "negative correlation." The most famous example from economics is price and quantity: increasing the price reduces the quantity demanded, and vice versa. Many examples of negatively related factors can be found in health care. For example, at the end of life, choices are often made between duration and quality of life.[1] Your workday is likely characterized by how hard you work (effort) and how long you work (hours), and these are often negatively related.[2]

Negatively related variables present a challenge when a leader's objective means the two variables must be multiplied to reach an objective (e.g., maximizing total revenue as the product of price times quantity, or optimizing workplace performance represented by productivity per hour times hours worked). In this article, we consider how leaders should choose optimal courses of action when they care about the product of two variables that are inversely related. We summarize our findings with a general rule involving a concept called elasticity and offer suggestions for applying it.

Motivating examples

Example 1: As a member of your organization's parking committee, you discover that its only objective is to bring in the most money possible. Some committee members think that charging lower rates will achieve this objective. They assume that more people will purchase parking if the price is lower. In contrast, others assert that charging higher rates is a good idea because, even if fewer people choose to park, the organization will make more money per car. Which group is correct? Do you want more cars (and charge less per car) or a greater charge per car (and be less sensitive to fewer cars parking)?

Example 2: Your pet, Fluffy, is getting older and starting to experience severe health issues. The veterinarian has options to help your pet live longer. However, each option also results in a lower quality of life for Fluffy. Some family members want Fluffy to live as long as possible, even if that means her quality of time is lower. Others feel Fluffy should enjoy a high-quality life, even if it is at a reduced duration. If you want to give Fluffy the highest quality-adjusted life years (defined as the product of quality of life and length of life), which option should you choose?

Example 3: At work, you wonder, should you work a few hours, putting in a lot of effort with high levels of concentration, or should you work many hours at a more measured pace, without expending too much effort (conserving your energy for the marathon day)? You can work longer if you do not focus as much, but you may not get as much accomplished. If you work hard, you will not be able to work as long. What's the optimal combination of productivity and hours worked so that you get the most done, given that these factors are negatively correlated?

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When things go in opposite directions

An important aspect of these examples is that the two key variables move in opposite directions. If you raise the price of parking, demand decreases. If you lengthen Fluffy's life, she will experience a lower quality of life during those extra days. If you increase your concentration and effort per work hour, the number of hours you can work at that level decreases. Figure 1 illustrates such negative relations between two key variables.

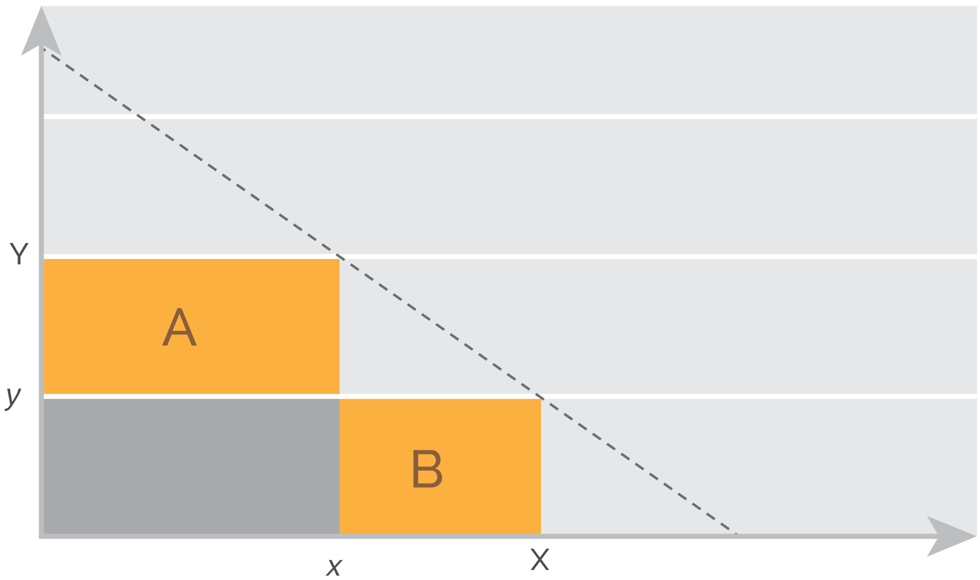

Figure 1. Illustrating negatively related factors and their products

Figure 1 shows the higher value (Y) on the vertical axis above the lower value (y). Likewise, the higher value on the horizontal axis (X) is farther from the origin than the lower value (x). The negatively sloped dashed line shows that with higher Y you get lower x, and with lower y you get higher X. As a reminder, the objective is to maximize the area represented by the product, or multiplication, of the two variables (shown as shaded rectangles). An optimal decision is the course of action that yields the biggest shaded area.

In Figure 1, it is clear that the area denoted by xY (A) and the area denoted by Xy (B) overlap (in the "dark grey" area). Geometrically, $xY = Xy$ if rectangle A has the same area as rectangle B. We can calculate when this occurs as we know that area $A = (Y - y)x$ and area $B = (X - x)y$.

Setting the two areas equal, we see that occurs when $(Y - y)x = (X - x)y$, requiring $(Y - y)/y = (X - x)/x$. Another way to write this condition is that the two options (x, Y and X, y) provide the same payoff when the percentage change in x ($\%\Delta x$) equals the percentage change in y ($\%\Delta y$). If increasing y by 10% leads to a decrease in x of 10%, there is no overall gain or loss. However, if increasing y by 10% leads to a decrease in x of less than 10%, there will be a gain (or a loss if the decrease in x is more than 10%). The upshot is that a beneficial change is determinable by how sensitive one variable is to changes in the other; if you can raise one variable by more than the other decreases (all in % terms), you will help your cause.

Elasticity: how sensitive one is to the other

Economists use elasticity to refer to how sensitive a variable is to changes in another variable. For example, high elasticity means that when one variable changes a little (in percentage terms), the other variable changes a lot (in percentage terms). In contrast, low elasticity means a large percentage change in one variable will be accompanied by a small percentage change in the other.

Janko and Kakar[3] studied the relation between per capita health care expenditures and per capita income using Canadian provincial data spanning 40 years from 1981 to 2020. They estimated income elasticities in the 0.11–0.16 range, meaning if a province's per capita income increases by 10%, its per capita health care expenditures are expected to increase by 1.1–1.6%. Provincial health care expenditures are relatively insensitive to per capita income, i.e., the elasticity, calculated as percentage change (Δ%) in health care spending over Δ% income is less than 1.

Although the dashed line in Figure 1 has a constant slope (because it is a line), it does not have a constant elasticity. This makes sense, as elasticity is about percentage change. Specifically, elasticity is the ratio of one percentage change over the other. If you choose a point on the dashed line where there is a big Y and a small x and you change Y, the %ΔY will be small (because Y is big), but the %Δx will be large (because x is small). Likewise, if you select a small y and a big X and you change y, the %Δy will be big (because y is small), but %ΔX will be small (because X is big).

When we considered the relative merits of (x, Y) versus (X, y) in Figure 1, we determined that the two were equivalent when area A = area B. If area A > area B then (x, Y) is better than (X, y) and vice versa. These conditions can be summarized in terms of elasticity, and their decision implications are shown in Table 1.

Table 1: Adjustments to make to increase the value of x times y

Current condition			Objective
Percentage change in x ($\%\Delta x$)	Percentage change in y ($\%\Delta y$)	Elasticity $\lvert \%\Delta x / \%\Delta y \rvert$	Maximize the product of the two variables
Smaller	Bigger	< 1	Increase x, decrease y
Bigger	Smaller	> 1	Decrease x, increase y
Same		1	Already optimal

Note: For the assumed negative relation, elasticity is taken as an absolute value so $\lvert \%\Delta x / \%\Delta y \rvert$ is referred to as the y elasticity of x.

How to apply these concepts

In the motivating examples above, the goal was to maximize the product of two variables (i.e., Y times X). The two variables are assumed to be negatively related as shown in Figure 2 with a dashed line (e.g., related by the formula $Y = aX + b$ where $a < 0$ and $b > 0$).

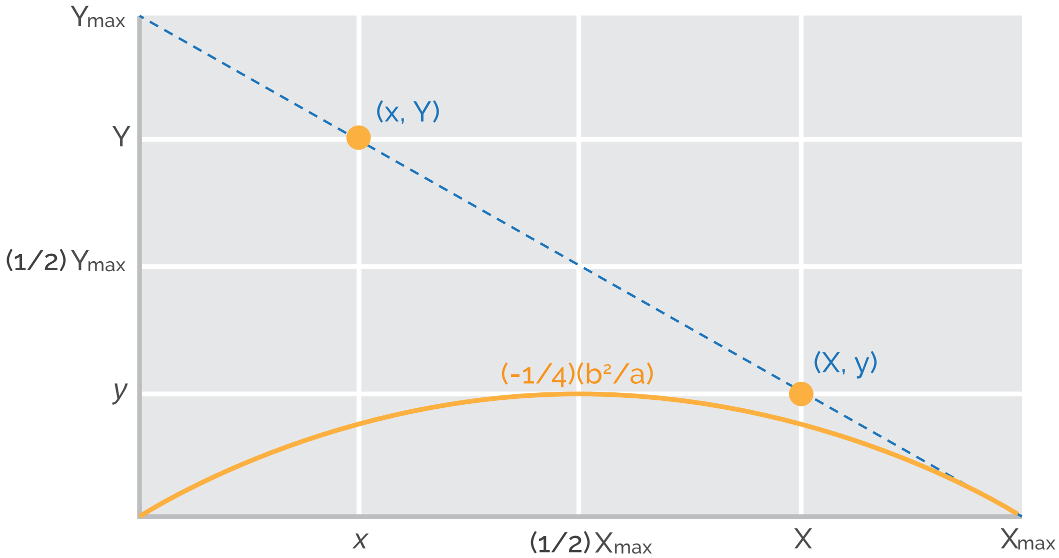

Figure 2. Trade-offs along the dashed lines yield outcomes along the solid curved line

Since everyone loves parking, let's assume that the price set for parking is on the vertical axis (Y variable) and the number of parking spots used is on the horizontal axis (X variable). What price do we set for parking to make the most money? Total revenue is the product of price and quantity, and different total revenue amounts are illustrated by the curved solid line. To make things simple, assume you see no cars parked when the price is $40 and the lot is completely full with 40 cars when the price is $0. Based on these assumptions, we believe $Y_{max} = 40$ and $X_{max} = 40$.

The curved line crests at a value of $X_{max}/2 = 40/2 = 20$ cars. According to the dashed line, we can get 20 cars when we charge $Y_{max}/2 = \$40/2 = \$20$. In other words, to make as much money as possible from parking revenue, it is best to charge \$20 which you expect will result in the purchase of 20 parking spaces, leaving half the lot empty.

Discussion

In the example above, it might seem strange that it is optimal to choose a price so high that only half of the parking lot is filled. The dashed line in Figure 2 clearly shows that we could increase demand by lowering the price. Why not lower the price and entice more people to park? The answer to this question relates to the objective of maximizing total revenue (i.e., the product of the two negatively related variables). In the first paper of this series on health economics, we stressed the crucial role "objective" plays in determining optimal choices, noting that simply focusing on one outcome to monitor (such as total revenue) may produce overall inefficiency according to a more comprehensive set of objectives (e.g., total revenue and affordable parking).[4] In this case, revenue maximization and universal access are at odds, and when the only stated objective is to make money, this is accomplished when the lot is half full (as indicated by the peak of the curved line in Figure 2).

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… revenue maximization and universal access are at odds, and when the only stated objective is to make money, this is accomplished when the lot is half full …

In the real world, you do not have Figure 2. However, you can see cars parked in the lot as a result of the stated price. If you currently see 25% capacity being used based on a high price for parking, you can use the concept of elasticity to motivate the case for reducing the price of parking. A reduction in the high price enough to increase use is likely going to be a small percentage and result in a larger percentage increase in use. This type of thinking is illustrated in Figure 2. If you start at point (x, Y), you can make more total revenue by decreasing price from Y to $\frac{1}{2}Y_{max}$. In contrast, if you see a nearly full lot, the intuition is that you are starting at a point like (X, y). You will generate more total revenue by increasing the price from y to $\frac{1}{2}Y_{max}$. The elasticity is such that the increase in money per car overshadows the decrease in number of cars (due to the increased price).

The types of problems we have discussed are solved by thinking about the horizontal value that maximizes the objective (the curved line in Figure 2). In Figure 2, the curved line reaches its highest point when both the vertical and horizontal variables are half their largest values. The top of the curved line in Figure 2 occurs at ½X_{max} and this is achieved when ½Y_{max} is chosen. Any combination of X and Y along the dashed line in Figure 2 can be chosen, but only one combination results in maximizing the stated objective.

This type of thinking is useful whether we are considering productivity and hours worked, quality of life and length of life, or the price of parking and parking spaces filled. When thinking about elasticity, leaders should consider whether a 10% change in one variable (say price) would yield a greater (or less) percentage change in the other variable (say quantity). If the reaction will be large and unfavourable, this signals that leaders should avoid that direction. In contrast, if you can raise the price, increase quality of life, or raise productivity by 10% with a corresponding decrease of less than 10% in the other variable, that will help more overall than it hurts. The intuition from the concept of elasticity is that, typically, the variable that is relatively large should be reduced and the variable that is relatively small should be increased.

Conclusion

Leaders often face difficult trade-offs where, if they want more of something, they must accept less of something else. Faced with the objective of maximizing the product of two variables, the optimal choice can be counterintuitive. Sometimes, to make more revenue, price must be lowered. Other times, to get more done, one should increase one's productivity or effort and shorten work hours. Lastly, more quality-adjusted life years may be achievable by experiencing less length of life but at a higher quality of life.

> Lastly, more quality-adjusted life years may be achievable by **experiencing less length of life but at a higher quality of life.**

In the second paper of this series on health economics, we concluded that the best action is often not choosing the least extreme or the most extreme values, as minimization is different from maximization and both often differ from optimization.[5] The concept of elasticity suggests that if you are making choices near the extreme limits, you may do better by moving toward the middle.

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Math appendix (for those with trouble sleeping)

Let the equation for the downward sloping line in Figure 1 be $y = ax + b$ where $a < 0$ and $b > 0$. When $x = 0$, the y intercept is b. Call this value Y_{max} as it is the largest value for the variable on the vertical axis. When $y = 0$, the x intercept is $-b/a$. Call this value X_{max} as it is the largest value for the variable on the horizontal axis (because $b > 0$ and $a < 0$, we know the value $-b/a > 0$). Calculus shows that the largest value of the product of the vertical and horizontal variables is found when $x = -b/2a$ (by taking the derivative of ax2 + bx with respect to x and setting it equal to 0). When $x = -b/2a$, then $y = a(-b/2a) + b$ or $b/2$. Recall $Y_{max} = b$, so that means the optimal value of y is half the highest value it can be. Similarly, since optimal $x = -b/2a$, and the formula for X_{max} is $-b/a$, we can see that optimal x equals $X_{max}/2$. Thus, the biggest area is achieved when half of the highest value of X and half of the highest value of Y are chosen. This produces an area equal to $X_{max}Y_{max}/4$. Choosing an x value different from $x = -b/2a = X_{max}/2$ means that the area represented by the product xy is not maximized. Choosing a vertical value different from $y = b/2 = Y_{max}/2$ when $x = X_{max}/2$ means we are no longer on the line with equation $y = ax + b$. Therefore, the optimal values are $X_{max}/2$ and $Y_{max}/2$. The curved line in Figure 2 shows the product of the two variables. If we insert the optimal $X_{max}/2$ into the formula for total revenue, we get $½X_{max}$ $½Y_{max}$ which equals ¼ X_{max} Y_{max}. From earlier, we showed that $X_{max} = -b/a$ and $Y_{max} = b$, so the largest value for the product of the two variables is −b2/4a and occurs when $X_{max}/2$ and $Y_{max}/2$ are selected. This is shown in Figure 2 at the highest point of the curved line denoted as (−¼)(b2/a).

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